Suggested Solution

閱讀材料

Reading Materials

機會率

Probability

一件事件發生的可能性可用*機會率*表達。機會率是一個在 0 至 1 之間的數字，機會率越接近 0 即發生的可能性越小，機會率越接近 1 即發生的可能性越大。

The likelihood of occurrence of a particular event can be represented by a value known as *probability*. Probability is a value lies between 0 and 1, the more closer to 0 the event is less likely to occur, the more closer to 1 the event is more likely to occur.

一件事件的機會率可以以下的方式計算：

若結果是可數的：$\frac{\text{符合事件要求的結果數目}}{\text{可能出現的結果數目}}$

若結果是不可數的：$\frac{\text{符合事件要求的結果面積}}{\text{可能出現的結果面積}}$

The probability of an event can be calculated by the following formula:

$$\frac{\text{number of outcomes favourable to requirement of event}}{\text{number of possible outcomes in event}} \quad \text{(outcomes are countable)}$$

$$\frac{\text{area of outcomes favourable to requirement of event}}{\text{area of possible outcomes in event}} \quad \text{(outcomes are uncountable)}$$

Suggested Solution

例子 1：
Example 1:

投擲一粒公平骰子，求擲得偶數的機會率。
When a fair dice is tossed, find the probability that the number is even.

答案：
Answer:

事件要求：擲得偶數
Requirement of event: the number is even

可能出現的結果：1, 2, 3, 4, 5, 6
Possible outcomes in event: 1, 2, 3, 4, 5, 6

符合事件要求的結果：2, 4, 6
Outcomes favourable to the requirement of event: 2, 4, 6

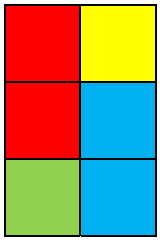
$$\text{機會率} = \frac{3}{6} = \frac{1}{2} = 50\%$$

$$\text{Probability} = \frac{3}{6} = \frac{1}{2} = 50\%$$

Suggested Solution

例子 2：

Example 2:



上圖的長方形由六個邊長為1 cm 的正方形組成，正方形內填上不同的顏色。
若在大長方形內隨意選擇一點，求該點是藍色的機會率。

The diagram above shows a rectangle composed of six squares with sides 1 cm, each square is filled by some colors.
If a point is randomly selected from the rectangle, find the probability that the chosen point is blue in color.

答案：

Answer:

每個正方形的面積為1 cm^2 。

The area of each square is 1 cm^2.

事件要求：選擇到的點是藍色

Requirement of event: The chosen point is blue in color

可能出現結果的面積：6 cm^2

Area of possible outcomes in event: 6 cm^2

符合事件要求結果的面積：2 cm^2

Area of outcomes favourable to the requirement of event: 2 cm^2

$$\text{機會率} = \frac{2\ \text{cm}^2}{6\ \text{cm}^2} = \frac{1}{3} = 33\tfrac{1}{3}\%$$

$$\text{Probability} = \frac{2\ \text{cm}^2}{6\ \text{cm}^2} = \frac{1}{3} = 33\tfrac{1}{3}\%$$

Suggested Solution

學校名稱：School Name:

第十七屆香港小學數學創意解難比賽

The 17th Hong Kong Mathematics Creative Problem Solving Competition for Primary Schools

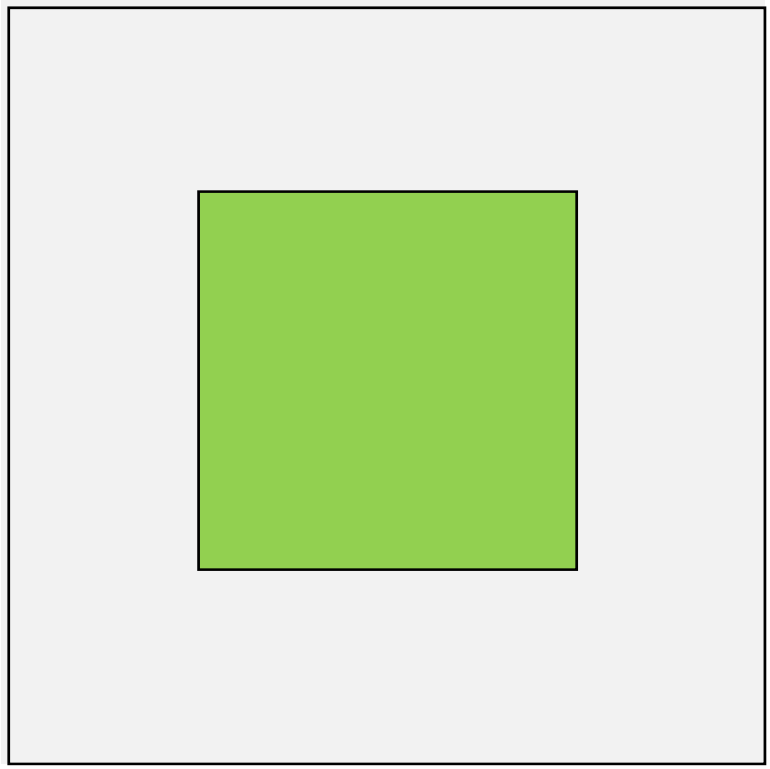
決賽

Final Event

Suggested Solution

創意解難題目　　(50 分鐘)
Creative Problem Solving Question　　(50 minutes)

1. 在下圖顯示一個邊長為10 cm 的白色正方形中間畫了一個邊長為5 cm 的綠色正方形。
In the figure below, a green square of side 5 cm is drawn inside another white square of 10 cm .



現在把一枚直徑為2 cm 的圓形硬幣投擲到正方形紙板上，假設全個硬幣都落在大正方形之內，若硬幣落在白色的部分或正方形的邊界上便算輸；若硬幣全個落在綠色部分便獲得一份獎品。
A circular coin of diameter 2 cm is tossed onto the square board. Assume the whole coin falls onto the large square. The game is loss if the coin falls on the white part or the side of the square; A prize is got if the whole coin falls on the green part.

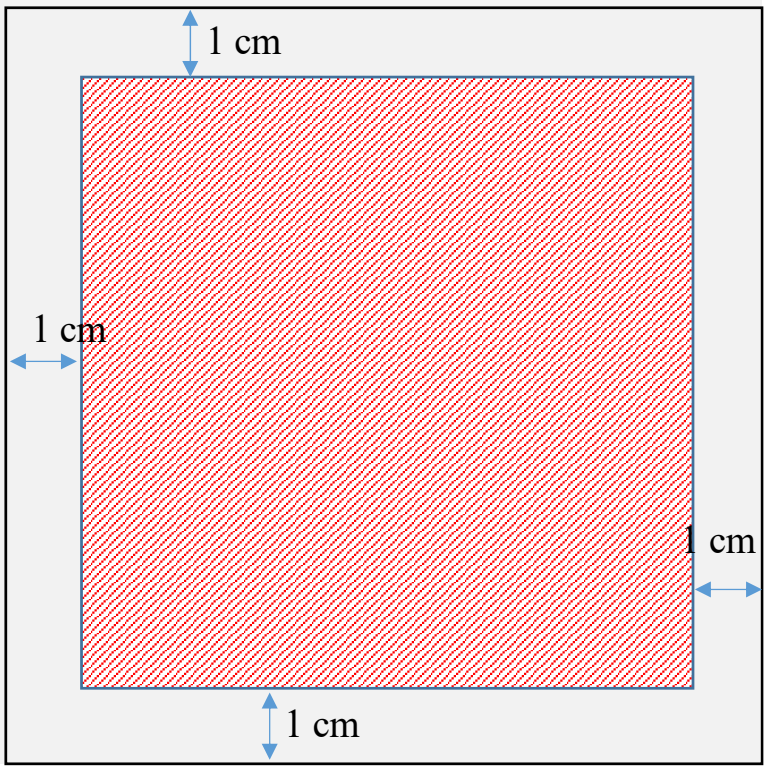
Suggested Solution

(a) (i) 在下圖中把硬幣圓心的所有可能位置塗黑，使得硬幣全個落在邊長為10 cm 的正方形內。

In the diagram below shade all the possible positions of the centre of a coin such that the whole coin falls inside the 10 cm square.

(3 分)

(3 marks)



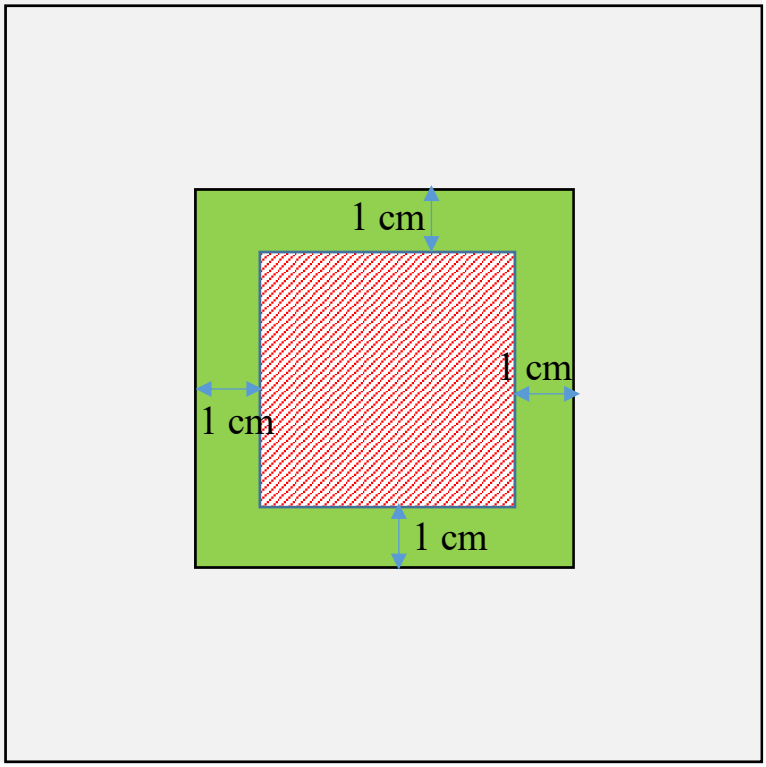
Suggested Solution

(ii) 在下圖中把硬幣圓心的所有可能位置塗黑，使得硬幣全個落在邊長為 5 cm 的正方形內。

Shade all the possible positions of the centre of a coin such that the whole coin fall inside the 5 cm square.

(3 分)

(3 marks)



(b) 求獲得獎品一份的機會率。

Find the probability of getting a prize.

(4 分)

(4 marks)

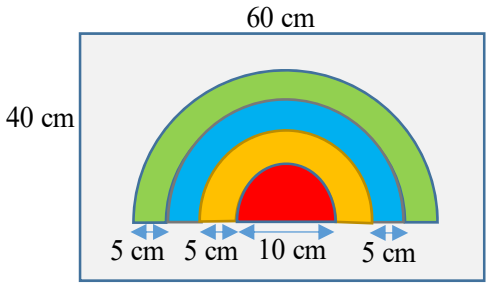
$$\text{Probability} = \frac{3\times 3}{8\times 8}$$
$$= \frac{9}{64}$$
$$= 14.0625\%$$

Suggested Solution

創意小學的校慶日有不同的籌款攤位遊戲，參與每項攤位遊戲前皆需付出一定費用去換取遊戲券，並根據遊戲的結果獲得不同銀碼的獎券。
There are different game stalls in the anniversary day of Creative Primary School. In each game stall, participants have to pay a certain amount of entry fee for a game ticket, and can win different amount of coupons according to the results of the game.

2. (a) 擲彩虹 Happy Rainbow

下圖為一張60 cm×40 cm 的長方形紙板，上面印了一道四色彩虹。中間紅色部分是一個直徑為10 cm 的半圓，其餘黃色、藍色及綠色彩虹的闊度皆為5 cm 。
The diagram below shows a rectangular cardboard of size $60\text{ cm}\times 40\text{ cm}$. A rainbow with four colors is printed on the cardboard. The middle red part is a semi-circle of diameter 10 cm. The widths of the yellow, blue and green parts are all 5 cm.



玩法：參加者每局需付出 n 元換取一張遊戲券，每張遊戲券可換得一枚直徑為 2 cm 的圓形硬幣。參加者需把硬幣擲到長方形紙板上，若硬幣落在白色的部分或同時落在多於一種顏色上便算輸；若硬幣全個落在綠色部分便獲得 10 元獎券、藍色部分便獲得 20 元獎券、黃色部分便獲得 30 元獎券、紅色部分便獲得 40 元獎券。
Game rule: Participant has to pay n dollars for a game ticket. Each game ticket is changed to a circular coin of diameter 2 cm. Participant throws the coin onto the rectangular cardboard. The participant loses if the coin falls on the white part or on any part covering more than one colors. The participant will win a 10 dollar coupon if the whole coin falls within the green part, a 20 dollar coupon if the whole coin falls within the blue part, a 30 dollar coupon if the whole coin falls within the yellow part and a 40 dollar coupon if the whole coin falls within the red part.

Suggested Solution

(i) 求出以下結果的機會率及所得的獎券銀碼。

Find the probability and the amount of coupon of each of the possible outcomes.

結果 Possible outcome	機會率 Probability	獎券銀碼 Amount of coupon
綠色 Green part	$\frac{19\times19\times\pi\div2-16\times16\times\pi\div2}{58\times38}$ $=7.48337633\%$	$10
藍色 Blue part	$\frac{14\times14\times\pi\div2-11\times11\times\pi\div2}{58\times38}$ $=5.345268807\%$	$20
黃色 Yellow part	$\frac{9\times9\times\pi\div2-6\times6\times\pi\div2}{58\times38}$ $=3.207161284\%$	$30
紅色 Red part	$\frac{4\times4\times\pi\div2}{58\times38}$ $=1.140324012\%$	$40

(5 分)

(5 marks)

(ii) 假設參加者隨機拋出硬幣，及都能把硬幣整個擲到紙板上。求出，並解釋參加者輸掉一局的機會率。

Assume that the participant randomly throw the coin, and the whole coin falls onto the cardboard. Find and explain the probability that the participant loses a game.

(5 分)

(5 marks)

Probability

$=1-7.48337633\%-5.345268807\%$

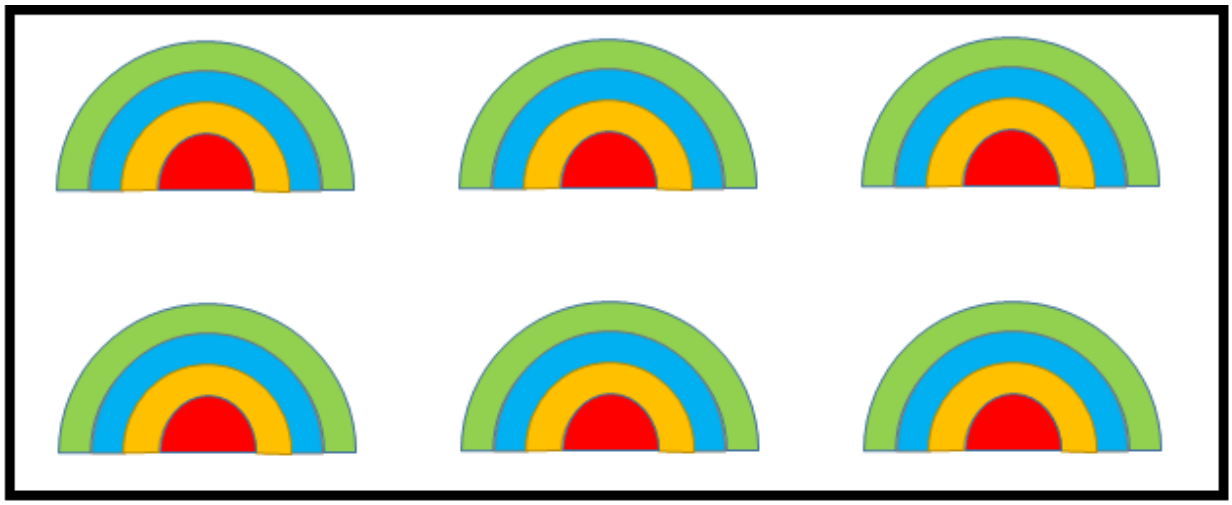
$-3.207161284\%-1.140324012\%$

$=82.82386957\%$

Suggested Solution

(iii) 主辦單位稍後把以上的彩虹複製成六個，並如下圖印製在一塊 180 cm×80 cm 的長方形紙板上。有參加者認為因為彩虹的數目多了，使得中獎的機會大了。你同意嗎？試解釋。

The organiser makes six copies of the above rainbow and prints on a cardboard with dimensions 180 cm×80 cm as shown in the diagram below. A participant thinks that the probability of getting a prize is larger because the number of rainbows is increased. Do you agree? Explain Briefly.



(10 分)

(10 marks)

No. Because the area of the possible positions of the centre of the coin increased at the original boundaries between adjacent rectangles, but the area of the favourable positions to get prizes is unchanged.

Therefore, the overall probability to get prizes becomes smaller.

Suggested Solution

(b) 主辦方提出參加者能以 2 張遊戲券，便可獲得直徑 1 cm 的圓形硬幣；
The organiser suggests that the participant can use 2 game tickets to exchange for a circular coin of diameter 1 cm

(i) 試解釋參加者使用 2 張遊戲券會使中獎的機會率增加嗎？
Explain whether the probability of winning a prize will increase if the participant use 2 game tickets.

(ii) 試解釋這個設定是否對參加者公平。
Please explain whether this suggestion is fair to the participant.

(10 分)
(10 marks)

(i)

Probability

$$= \frac{(19.5^2 - 15.5^2 + 14.5^2 - 10.5^2 + 9.5^2 - 5.5^2 + 4.5^2) \div 2 \times \pi}{59 \times 39}$$

$$= 21.86212619\%$$

Therefore, the probability of winning a prize will increase.

(ii)

The original probability of winning a prize is 17.17613043%.

The probability of winning a prize when playing two games

$$= 1 - (1 - 17.17613043\%)^2$$

$$= 31.4020662\%$$

$$> 21.86212619\%$$

Doubling the number of game tickets will not increase the winning probability

This game is unfair to the participant

Suggested Solution

(c) 文件夾內有一個硬幣及一張 A4 紙，當中印有兩個分別為 20 cm×10 cm 及 10 cm×5 cm 的長方形和一個 10 cm×10 cm 的正方形。試以這些圖形拼貼組合(可重覆使用)設計出「C」、「P」、「S」三個圖案，用來作為一個投擲硬幣遊戲的底板。

遊戲結果須包括大獎、中獎及細獎，獲得獎品的機會率須要大於 50%。

(你可以把長方形及正方形剪裁或摺疊，但不可用間尺作量度工具。)

(i) 在 A3 紙上劃出你的設計；

(ii) 在下面指定位置寫出遊戲的規則；

(iii) 在下表計算出獲得大獎、中獎及細獎的機會率。

There is a coin and an A4 sized paper in the folder. Two rectangles of sizes 20 cm×10 cm and 10 cm×5 cm, one square of size 10 cm×10 cm are printed on the paper.

Use the rectangles and square (you may duplicate and combine them) to design figures of shapes “C”, “P”, “S” as the board of a game of throwing coins.

The game result should include Big Prize, Middle Prize and Small Prize. The probability of winning prizes should be larger than 50%.

(You may crop and fold the rectangles and square, but you are not allowed to use a ruler as a measuring tool.)

(i) Draw your design on the A3 sized papers;

(ii) Write down the rules of the game in the space provided;

(iii) Calculate the probabilities of getting a Big Prize, a Middle Prize and a Small Prize in the table below.

(ii) 遊戲規則 Game Rules
Factors to be considered: (i) The corresponding parts for different prizes (ii) The number of tosses of the coin (iii) The management when part of the coin lies on the edge of the A3 sized paper (iv) Other reasonable creative rules

Suggested Solution

(iii) 獲得各獎品的機會率

Probabilities of getting each prize

大獎 Big Prize	中獎 Middle Prize	細獎 Small Prize

The area of each part should be calculated in detail.

The area of the favourable region and possible region should be clearly indicated.

(30 分)

(30 marks)

Suggested Solution